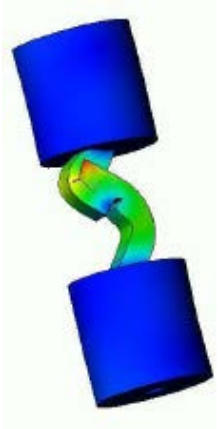
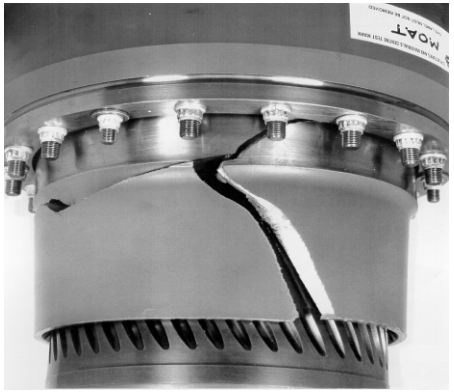
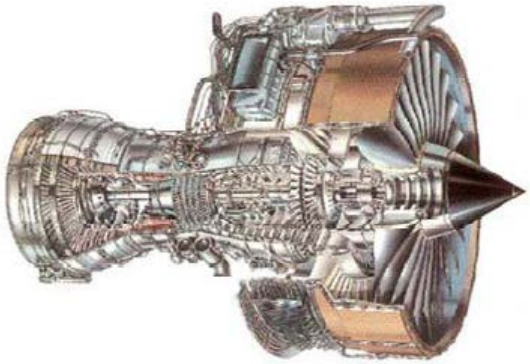
Exceptional service in the national interest







Modeling and Experimental Validation of a Pylon Subassembly Mockup with Multiple Nonlinearities

Students: Adam Bouma, Connor Ligeikis, Justin Shim

Mentors: Simone Manzato, Robert Kuether, Daniel Roettgen, Abdu Abdelkefi, Ed Habtour

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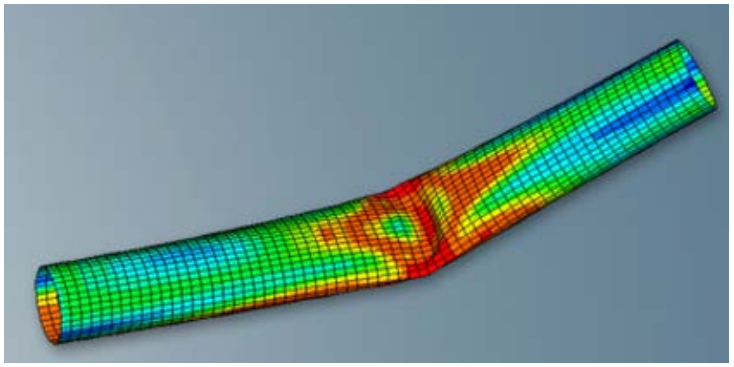
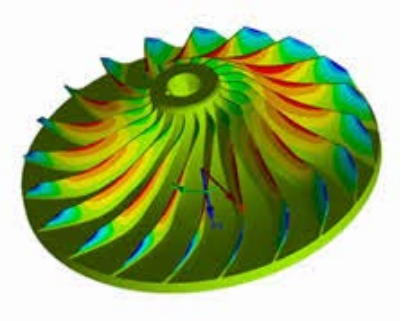
Outline

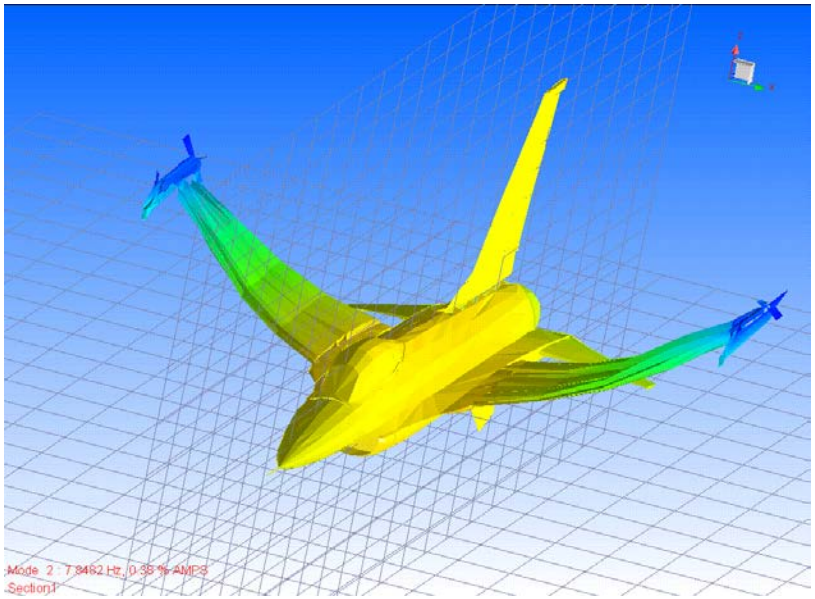
- Motivation
- Experimental Setup
- Linear Finite Element Model
- Low-level Modal Testing
- Nonlinear Testing and System Identification
- Conclusions
- Future Work

Motivation

- Numerical and experimental solutions rely heavily on a linear assumption
- When nonlinearities are prominent, linear solutions prove to be inadequate

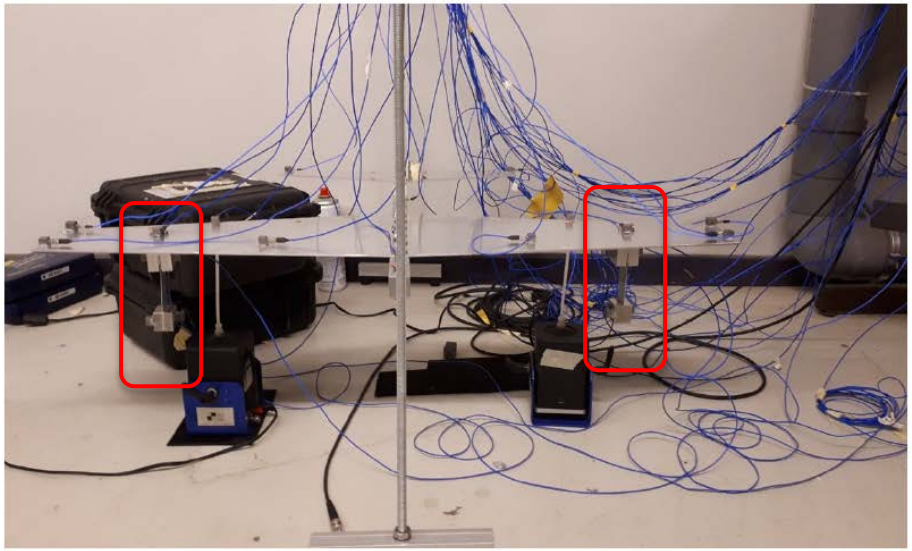


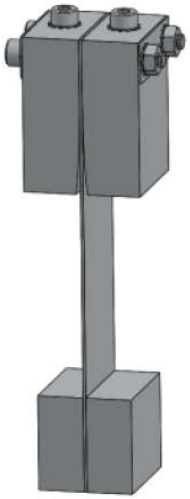




Motivation

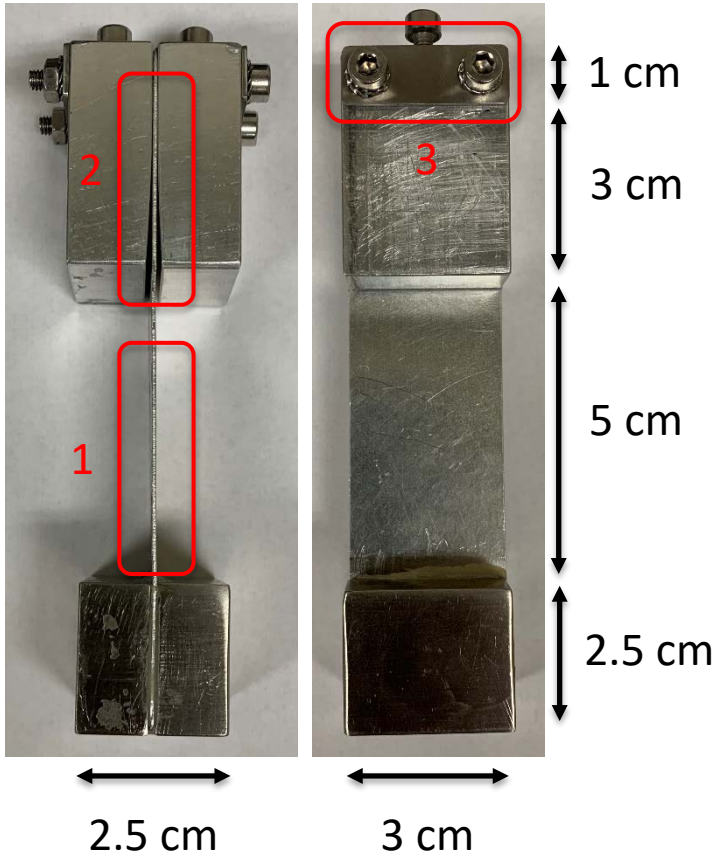
- A gap exists between academics and industrial communities
- Siemens Industry Software's demo aluminum aircraft, equipped with nonlinear pylon subassemblies
- From previous studies (Cooper et al., 2020), a more in-depth analysis on the pylon behavior was desired





Pylon

- Potential sources of nonlinearity
  1. Geometric nonlinearity of thin beam
  2. Variable stiffness, damping from contact
  3. Friction in the bolted connection

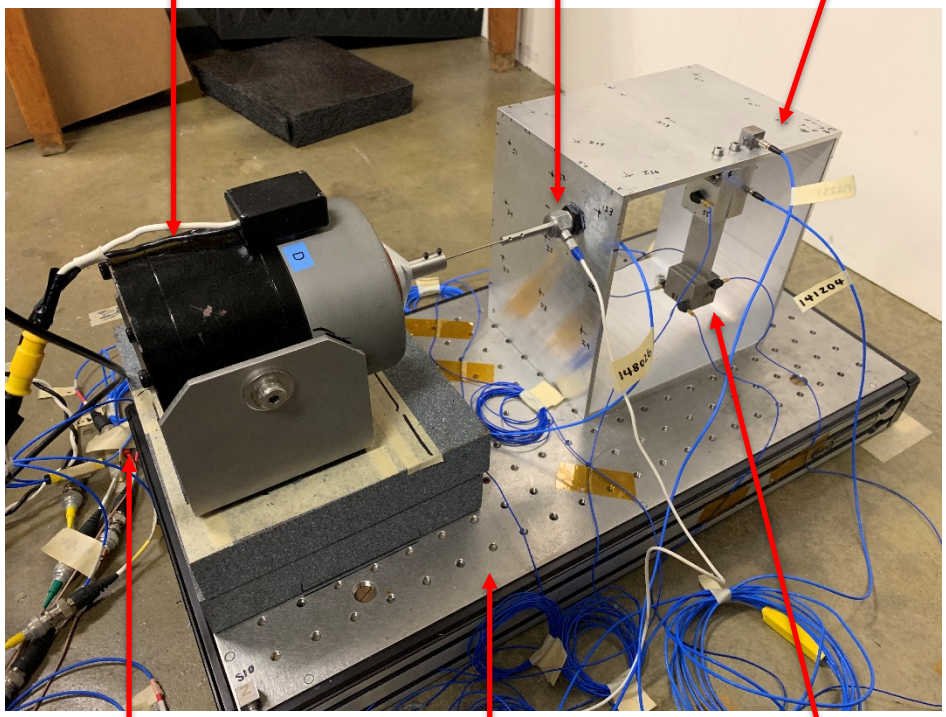


Experimental Setup

Shaker

Load Cell

Pylon Fixture

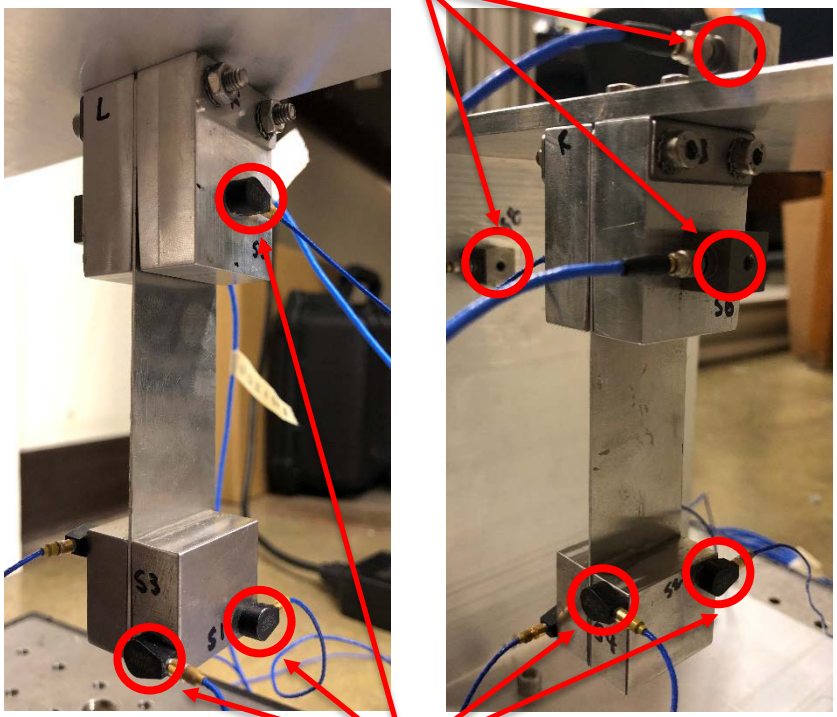


Connection to DAQ, Amplifier

Optical Breadboard

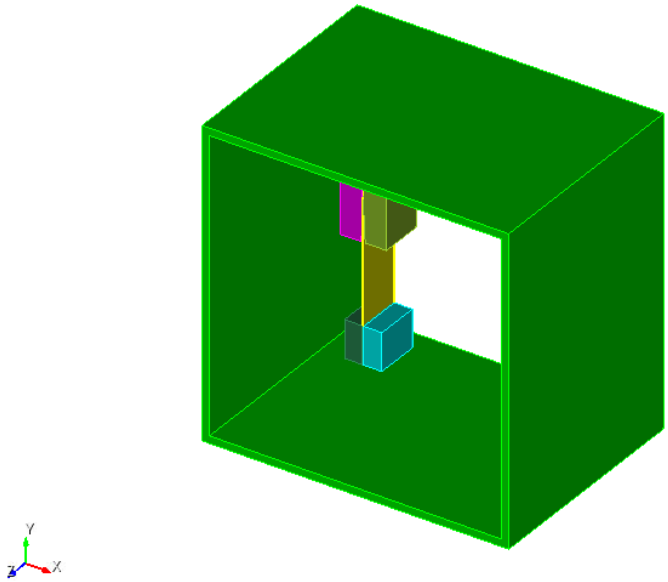
Pylon

Triaxial Accelerometers

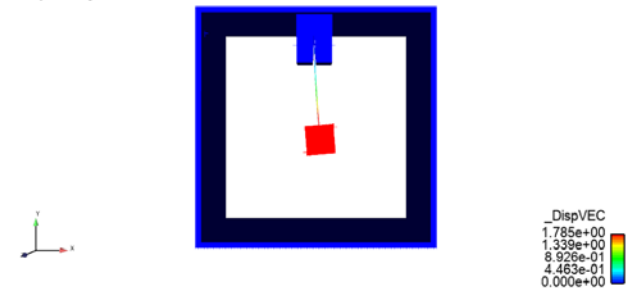


Uniaxial Accelerometers

Linear FEA Model



Frequency = 7.225



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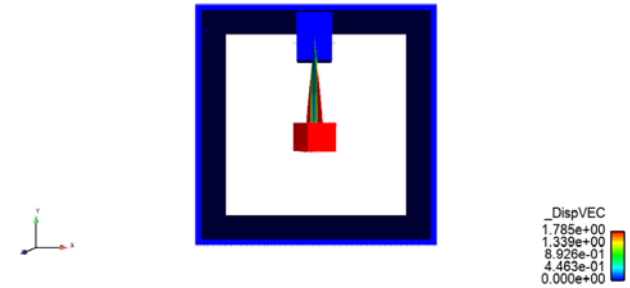
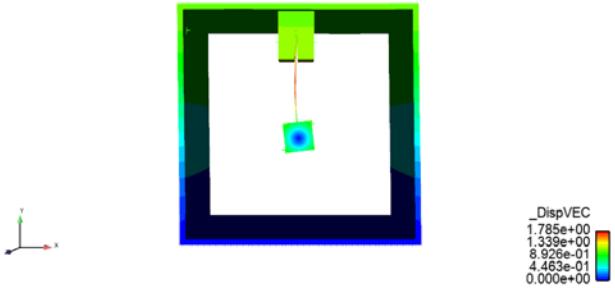
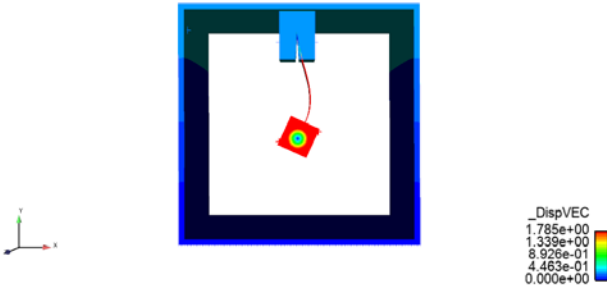


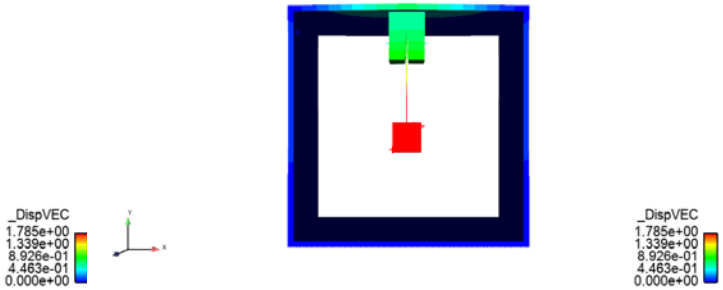
Frequency = 80.483



Frequency = 99.887



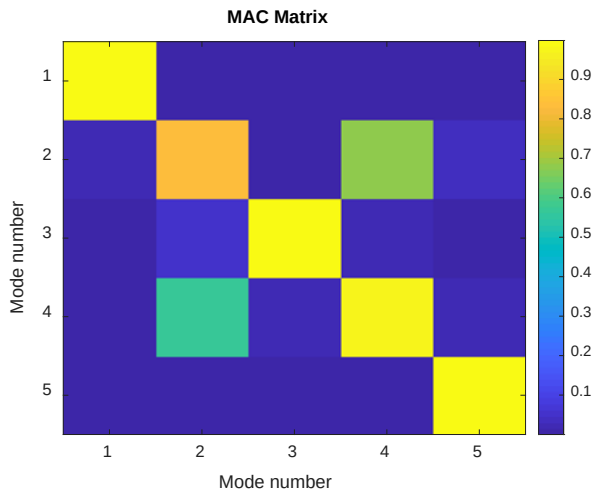
Frequency = 134.732

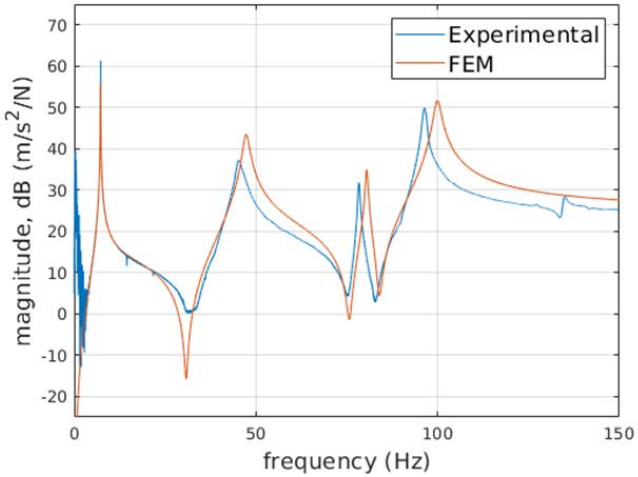


Low-level (Linear) Modal Testing and Analysis

- Combination of hammer and pseudo-random shaker testing used to identify linear natural frequencies, damping, and mode shapes

Mode	***1***	***2***	***3***	***4***	***5***
f_{exp} ***(Hz)***	7.25	45.79	78.07	96.30	134.75
f_{FEM} ***(Hz)***	7.22	47.28	80.48	99.89	134.73
% difference	**0.41**	**3.20**	**3.04**	**3.66**	**0.015**
ζ_{exp} ***(%)***	0.12	1.76	0.39	0.95	0.35

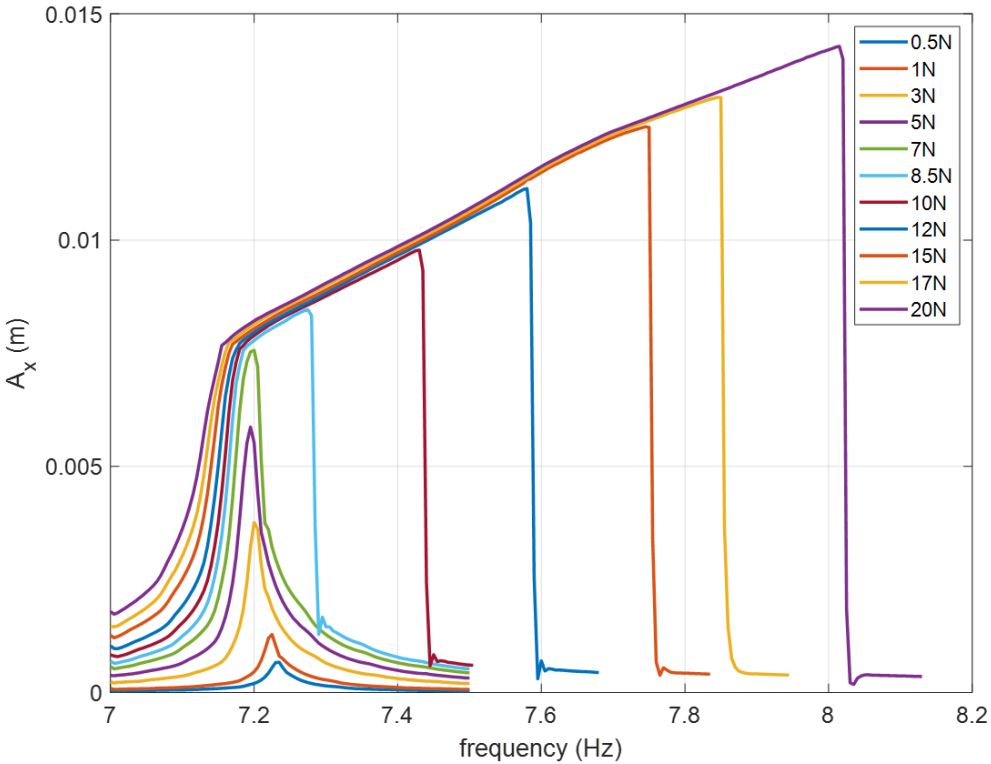




Nonlinear Testing Overview

- Two methods
  - Stepped sine tests at different force levels (CONCERTO)
  - “Flick tests” free decay (Hilbert, Zero Crossing)
- Goal
  - Identify frequency and damping vs. amplitude backbone curves
- Signal processing
  - Numerical integration of acceleration flick test time histories to get velocity and displacement
  - Bandpass filtering at 3 and 12 Hz cutoffs to isolate first mode response
- Cables had noticeable impact on level of damping in system
  - Tests were conducted with different numbers of cables attached to pylon to study this effect

Stepped-Sine Sweeps



CONCERTO Method

- **CO**de for **N**onlinear identifi**C**ation from m**E**asured **R**esponse to vibrati**O**n

  A. Carrella, D.J. Ewins (2010)

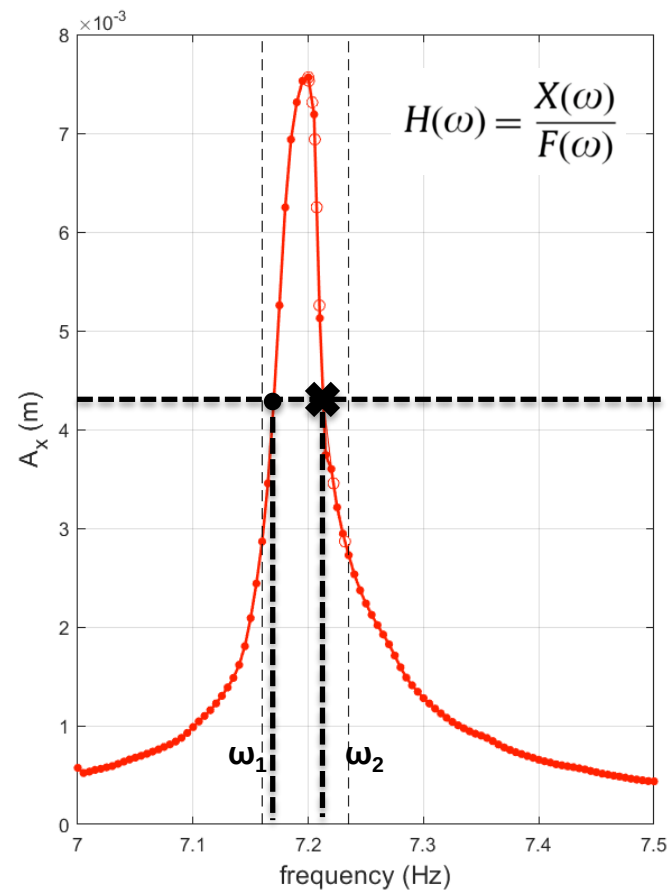
$$m\ddot{x} + c(X)\dot{x} + k(X) = F_0 \sin(\omega_e t)$$

1. Determine two frequencies on either side of the resonance with equal amplitude

2. $$H = \frac{X(\omega)}{F(\omega)} = \frac{A_r + jB_r}{\omega_n^2 + \omega_1^2 + j\eta_r\omega_n^2} = R_1 + jI_1$$
   $$H = \frac{X(\omega)}{F(\omega)} = \frac{A_r + jB_r}{\omega_n^2 + \omega_2^2 + j\eta_r\omega_n^2} = R_2 + jI_2$$

3. $$\omega_n(X) = \sqrt{\frac{(R_2 - R_1)(R_2\omega_2^2 - R_1\omega_1^2) + (I_2 - I_1)(I_2\omega_2^2 - I_1\omega_1^2)}{(R_2 - R_1)^2 + (I_2 - I_1)^2}}$$

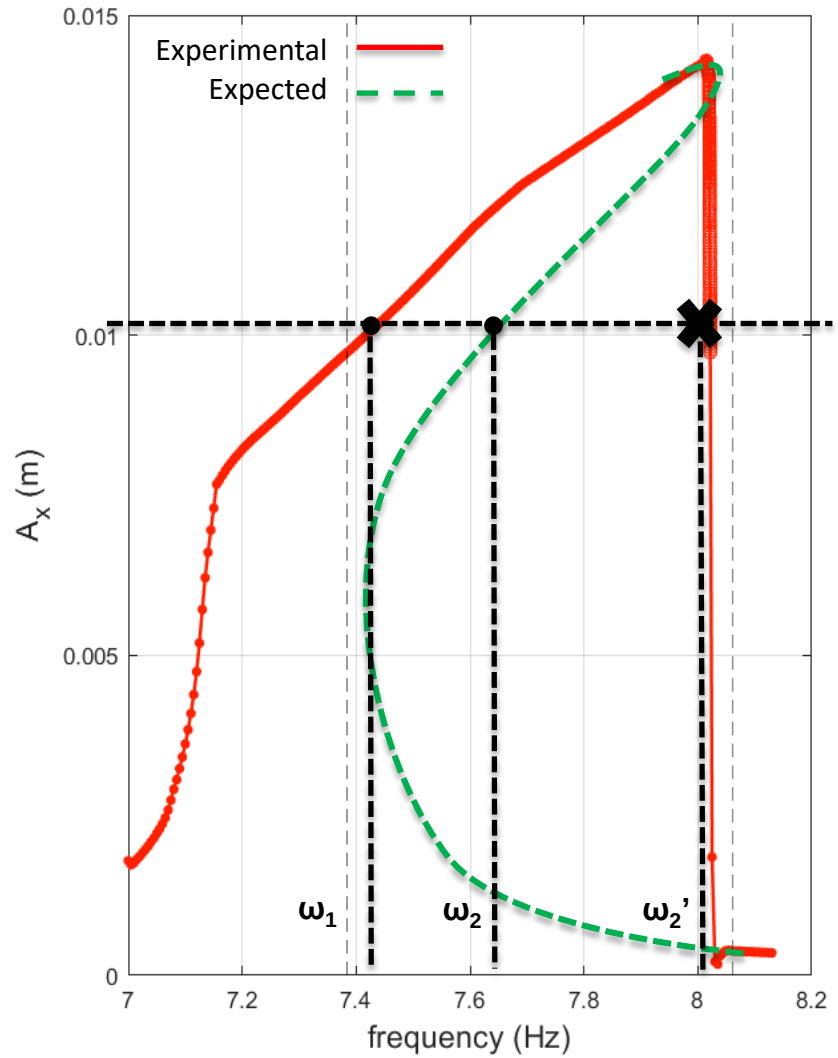
4. $$\zeta(X) = \left|\frac{(I_2 - I_1)((R_2\omega_2^2 - R_1\omega_1^2) + (R_2 - R_1)(I_2\omega_2^2 - I_1\omega_1^2)}{2\omega_n^2[(R_2 - R_1)^2 + (I_2 - I_1)^2]}\right|$$



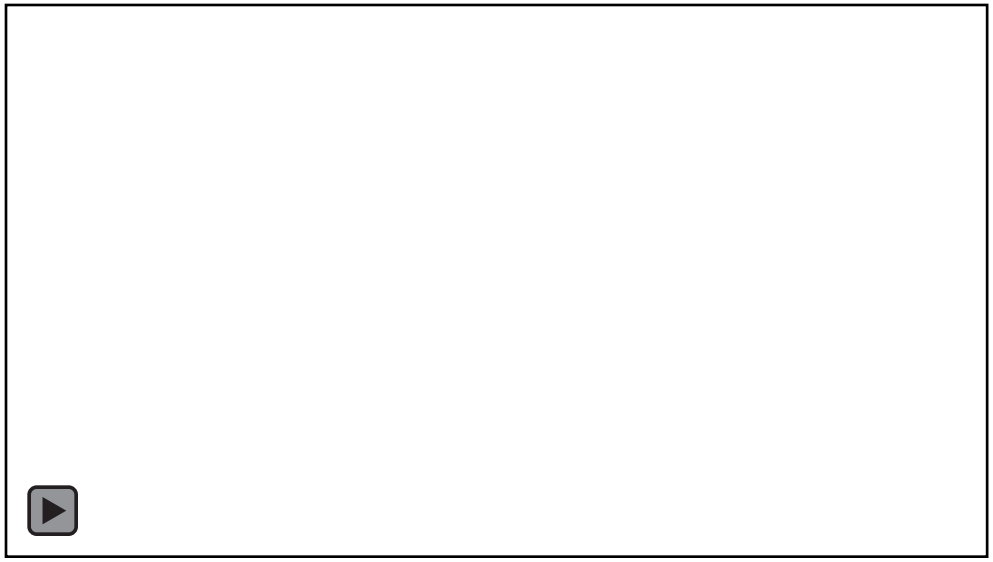
CONCERTO Method: Limitations

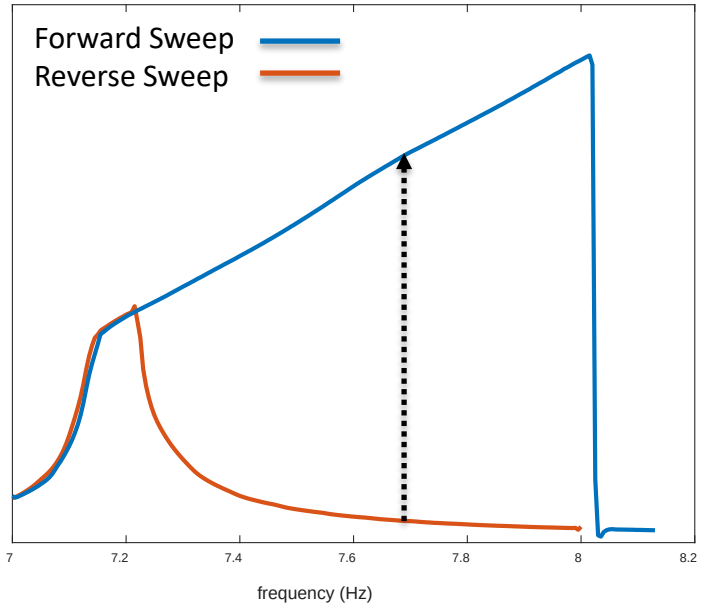
$$\omega_n(X) = \sqrt{\frac{(R_2 - R_1)(R_2\omega_2^2 - R_1\omega_1^2) + (I_2 - I_1)(I_2\omega_2^2 - I_1\omega_1^2)}{(R_2 - R_1)^2 + (I_2 - I_1)^2}}$$

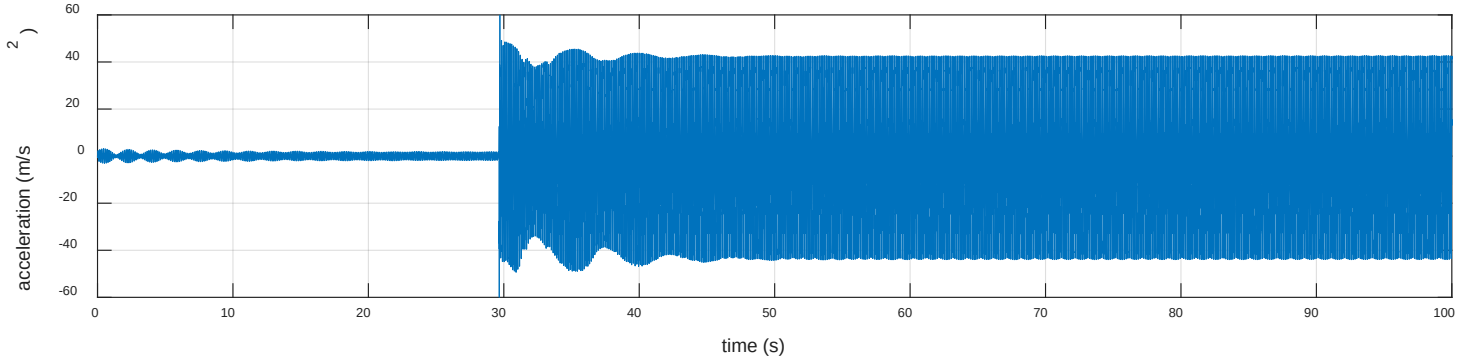
$$\zeta(X) = \left|\frac{(I_2 - I_1)((R_2\omega_2^2 - R_1\omega_1^2) + (R_2 - R_1)(I_2\omega_2^2 - I_1\omega_1^2)}{2\omega_n^2[(R_2 - R_1)^2 + (I_2 - I_1)^2]}\right|$$



Two Stable Solutions







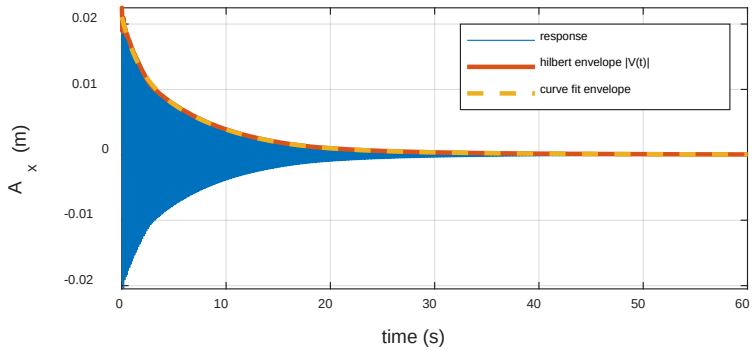
Hilbert Transform Approach

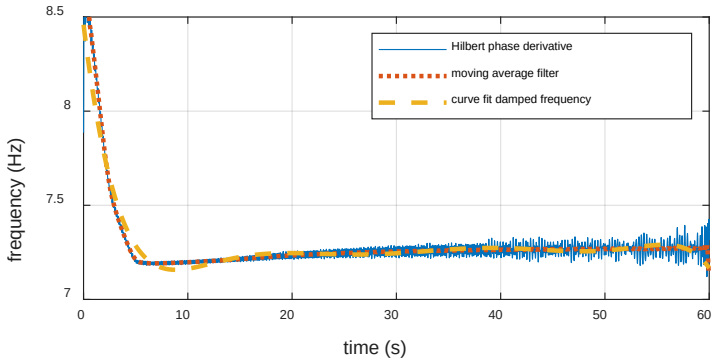
- **H. Sumali, R. Kellog** (2011)

1. Compute the Hilbert Transform on raw signal $v(t)$
   - Hilbert Transform: $\tilde{v}(t)$
   - Analytical Signal: $V(t) = v(t) + j\tilde{v}(t)$
2. The envelope of the ringdown is the magnitude of the analytical signal $|V(t)|$
   - Curve fit an nth order polynomial to the envelope
   - $|V(t)| = A_0 e^{-C(t)*t},\ C(t) = \zeta(t)\omega_n(t)$
3. The damped oscillation frequency is the derivative of the analytical signal's phase $\phi(t) = \tan^{-1}(\frac{\tilde{v}(t)}{v(t)})$
   - $\omega_d(t) = \frac{d\phi(t)}{dt}$

   a) Curve fit an mth order polynomial to the derivative

   b) Moving average filter on Hilbert phase derivative
4. Compute natural frequency and zeta values
   - $\omega_n(t) = \sqrt{\omega_d(t)^2 + C(t)^2}$
   - $\zeta(t) = -C(t)/\omega_n(t)$

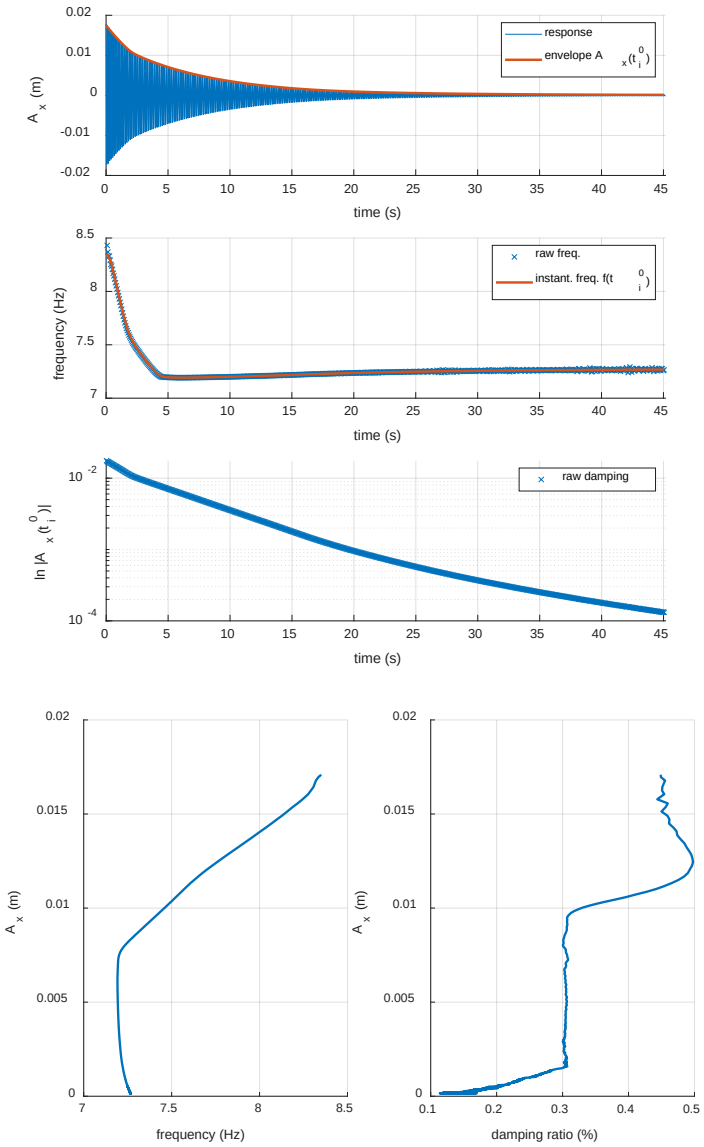




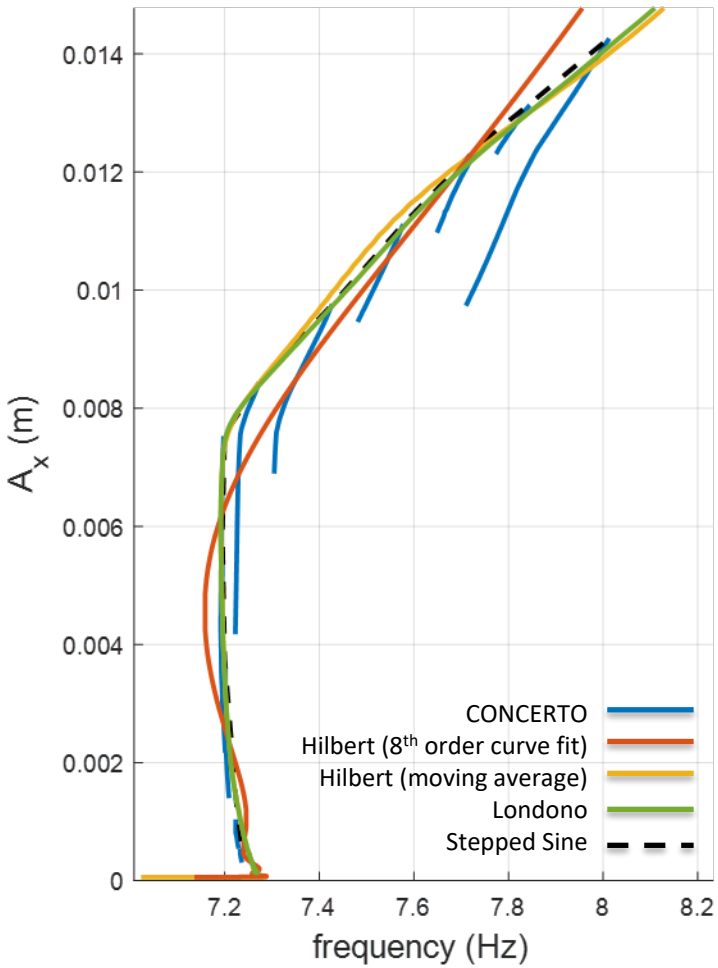
Zero Crossing Approach

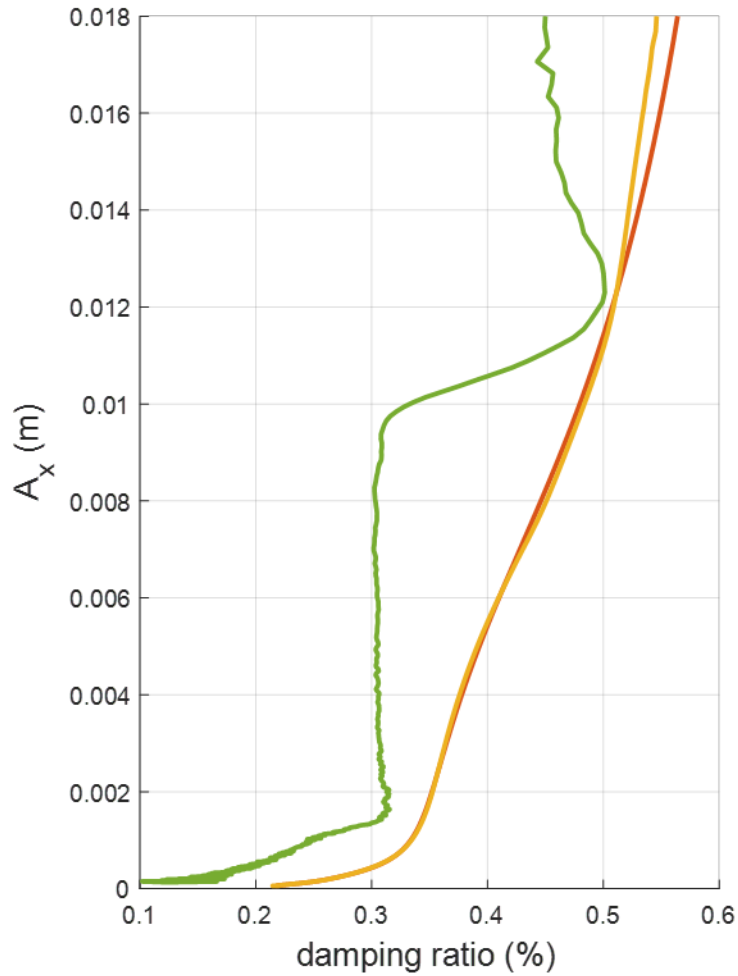
- **Londoño et al.** (2015)

1. Determine zero crossing times for $x(t)$: $\boldsymbol{t^0}$
2. Estimate instantaneous frequency for each t_i^0:
   $\hat{f}(t_i^0) = (t_{i+1}^0 - t_{i-1}^0)^{-1}$
3. Apply moving average filter to smooth $\hat{\boldsymbol{f}}$:
   $f(t_i^0) = \frac{1}{N}\sum_{j=0}^{N-1} \hat{f}(t_{i+j}^0)$
4. Determine max amplitude for each zero crossing time interval: $\hat{a}_i = \max_{\forall t|(t_i^0 \geq t \geq t_{i+1}^0)} |x(t)|$
5. Interpolate to get amplitude envelope at each t_i^0: $A_x(t_i^0)$
6. Find instantaneous damping ratio for each t_i^0:
   $\hat{\zeta}(t_i^0) = \frac{1}{\omega_0(t_i^0)(t_{i+1}^0 - t_{i-1}^0)}\left(ln(A_x(t_{i-1})) - ln\left(A_x(t_{i+1}^0)\right)\right)$
7. Apply moving average filter to smooth $\hat{\boldsymbol{\zeta}}$: $\zeta(t_i^0)$
8. Plot $f(t_i^0)$ vs. $A_x(t_i^0)$ and $\zeta(t_i^0)$ vs. $A_x(t_i^0)$

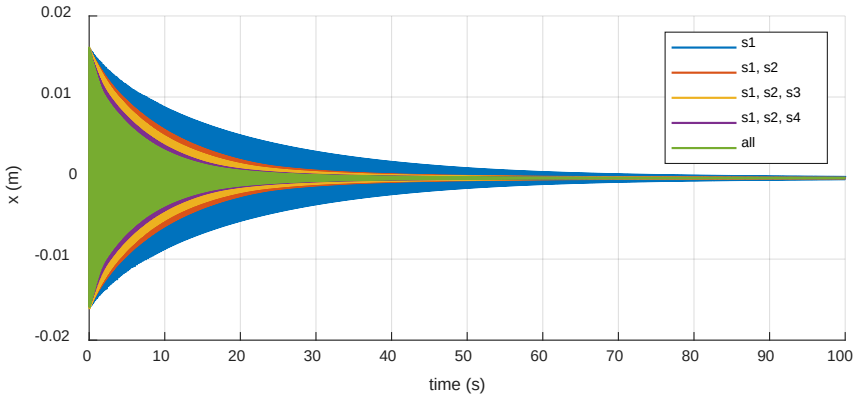


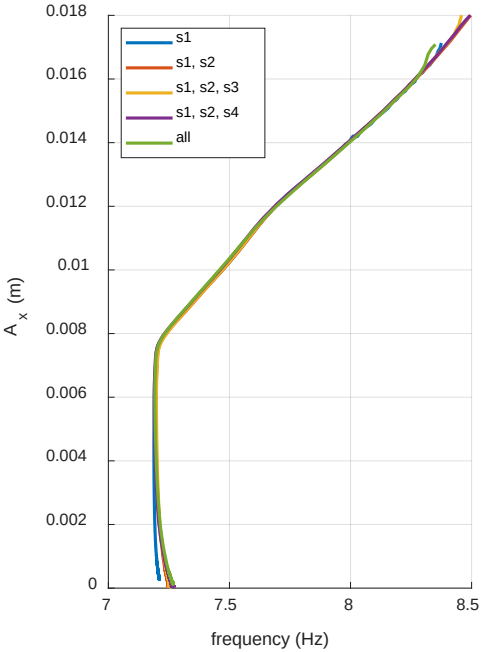
Comparison of Backbone Curves

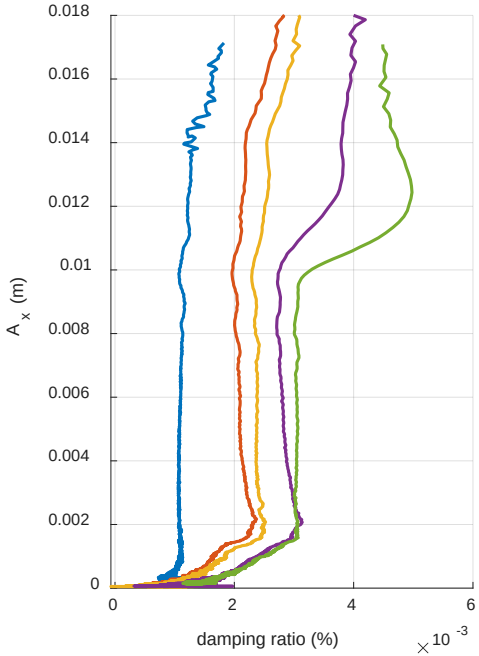


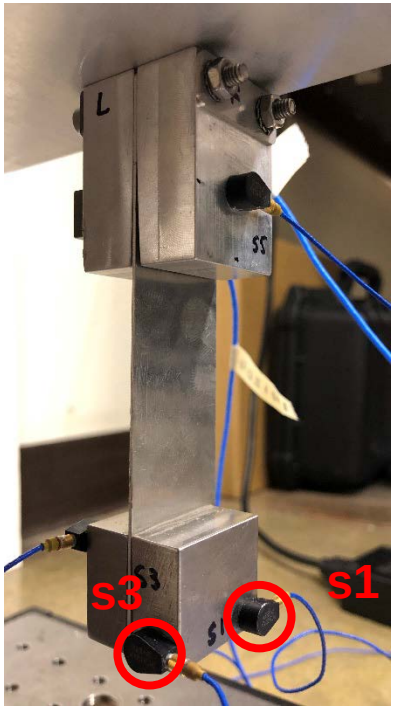


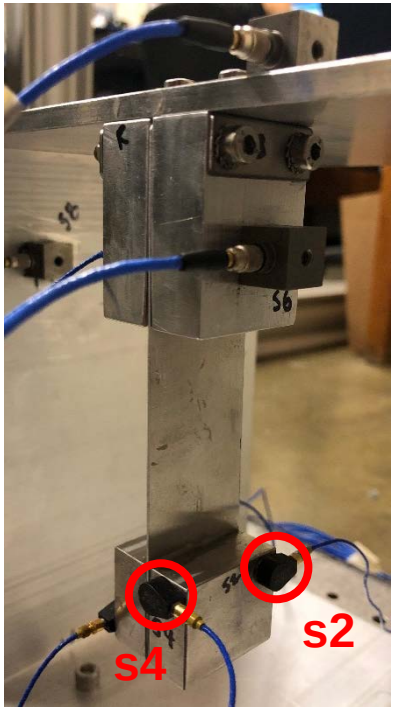
Effect of Accelerometer Cables











Nonlinear SDOF Model Identification (1 Cable)

$$\boldsymbol{m\ddot{x} + c_{nl}\dot{x} + k_{nl}x = f}$$

$$k_{nl} = k_3|x|^3 + k_2x^2 + k_1|x| + k_0 + k_{gap}\left(|x| - x_{gap}\right)$$

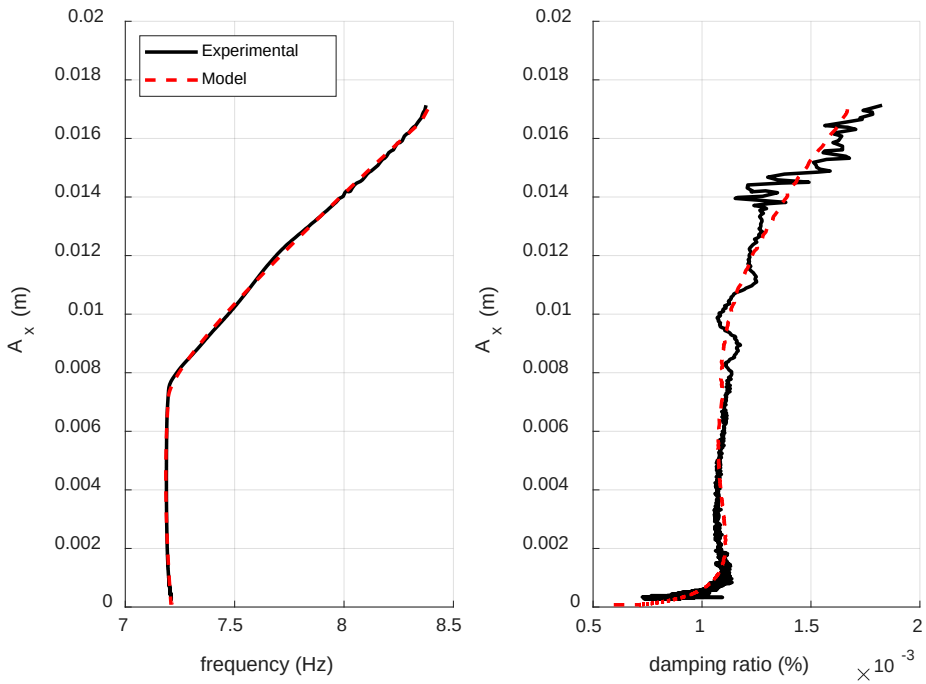
$$k_{gap} = 0 \text{ if } |x| < x_{gap}$$

$$c_{nl} = c_3|x|^{1.5} + c_2|x| + c_1\sqrt{|x|} + c_0$$

- Optimization performed using *fminsearch* MATLAB function to minimize difference in experimental and simulated backbones

$$k_{nl,opt} = \min_{k_{nl}}\left\|f_{exp}(A_x) - f_{sim}(A_x)\right\|$$

$$c_{nl,opt} = \min_{c_{nl}}\left\|\zeta_{exp}(A_x) - \zeta_{sim}(A_x)\right\|$$



Conclusions

- Pylon is highly nonlinear!
- Pylon exhibits weak initial softening behavior and then strong hardening behavior originating from contact
- Zero-crossing approach (Londoño et al.) seems to be most accurate and easiest to implement
- Difficult to say where damping originates (structural, friction, air, etc.)
- Sensor cables *significantly* affect pylon damping

Future Work

- Additional tests will be conducted to evaluate nonlinear SDOF model performance
- Exploring possible nonlinearities in other modes/coupling between modes and develop multi-DOF models
- Connecting linear airplane model with nonlinear SDOF model and studying combined system behavior
- *Strongly* recommend using lasers or DIC to get better damping estimates

Acknowledgements

- This research was conducted at the 2019 Nonlinear Mechanics and Dynamics Research Institute supported by Sandia National Laboratories and hosted by the University of New Mexico.

- Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA-0003525.

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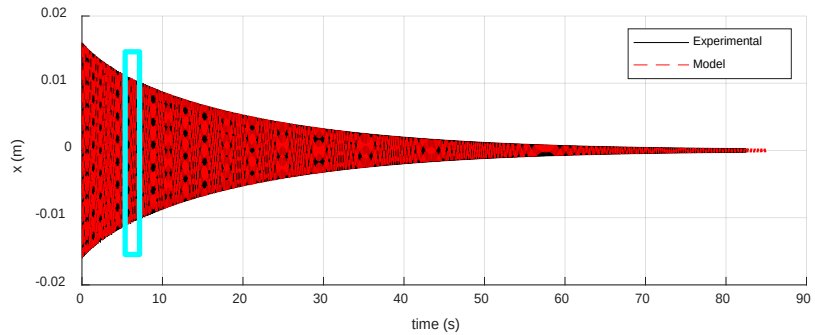
Thank you!

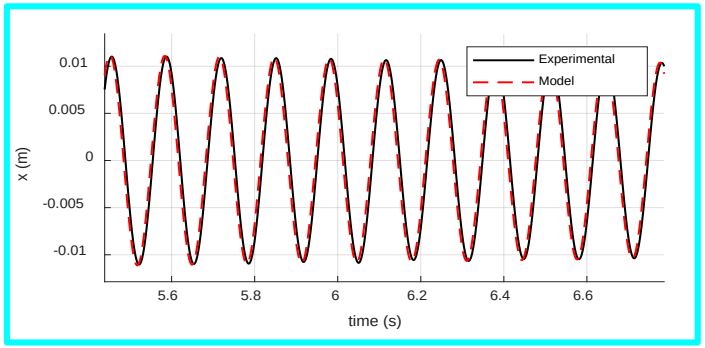
Nonlinear SDOF Model Performance

$$m\ddot{x} + \left(c_3|x|^{1.5} + c_2|x| + c_1\sqrt{|x|} + c_0\right)\dot{x} + \left(k_3|x|^3 + k_2x^2 + k_1|x| + k_0 + k_{gap}\left(|x| - x_{gap}\right)\right)x = f$$

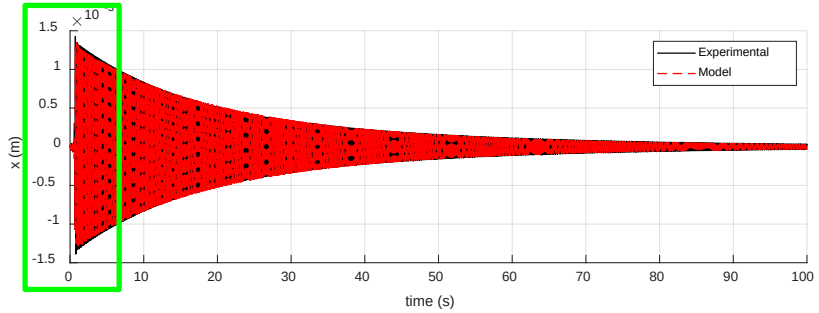
m	c_0	c_1	c_2	c_3	k_0	k_1	k_2	k_3	k_{gap}	x_{gap}
0.141 (kg)	**0.00564** (Ns/m)	**0.753** (Ns/m$^{1.5}$)	**-18.98** (Ns/m^2)	**140.9** (Ns/m$^{2.5}$)	**289.4** (N/m)	**-957.9** (N/m^2)	**90821** (N/m^3)	**5113296** (N/m^4)	**11440** (N/m^2)	**0.00723** (m)

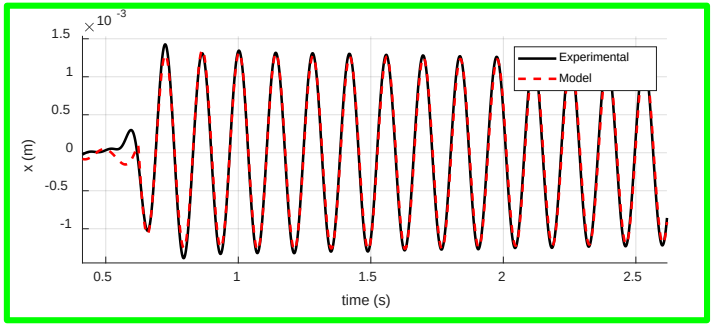
Free Decay:

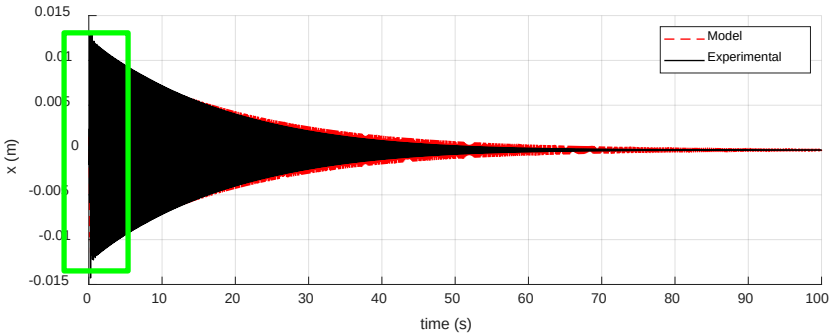
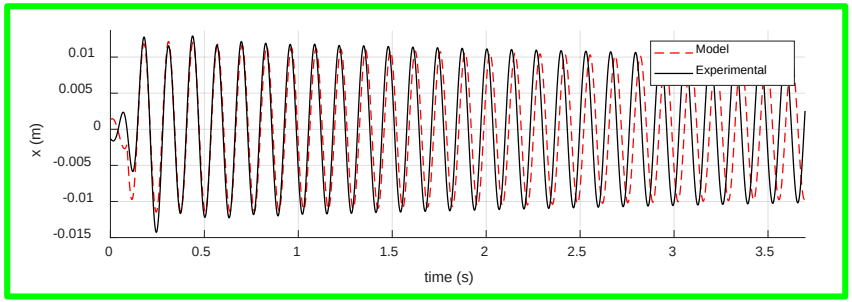
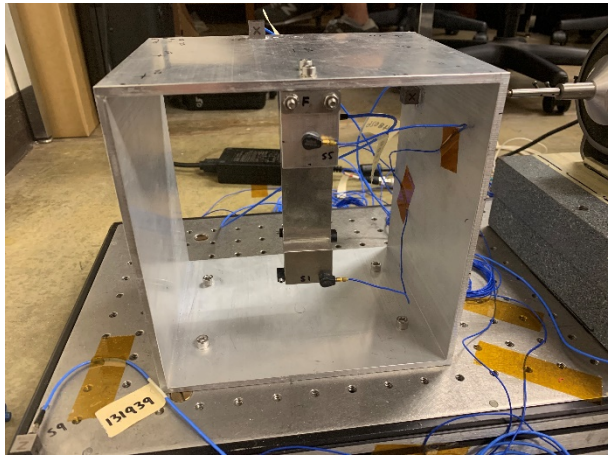
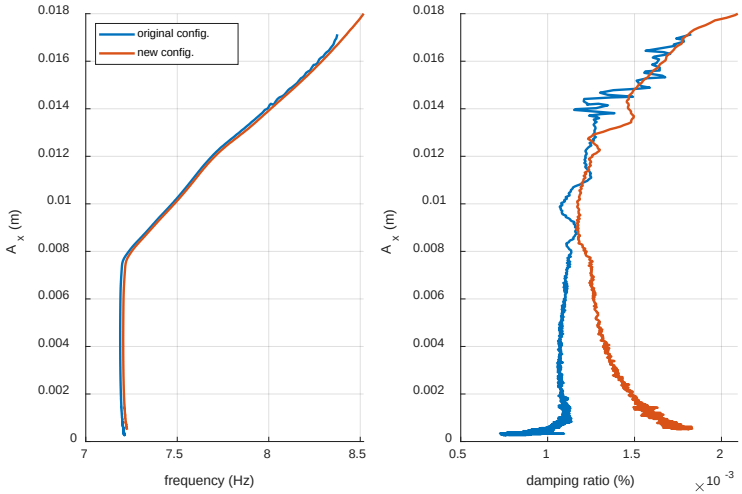




Low-level (~4 N peak) hammer input:





Nonlinear SDOF Model Performance (high level)

~50 N peak hammer input